5d Seiberg-Witten curve through toric-like diagram

2015-01-28

YongPyong-High1 2015

Joint Winter Conference on Particle Physics, String and Cosmology

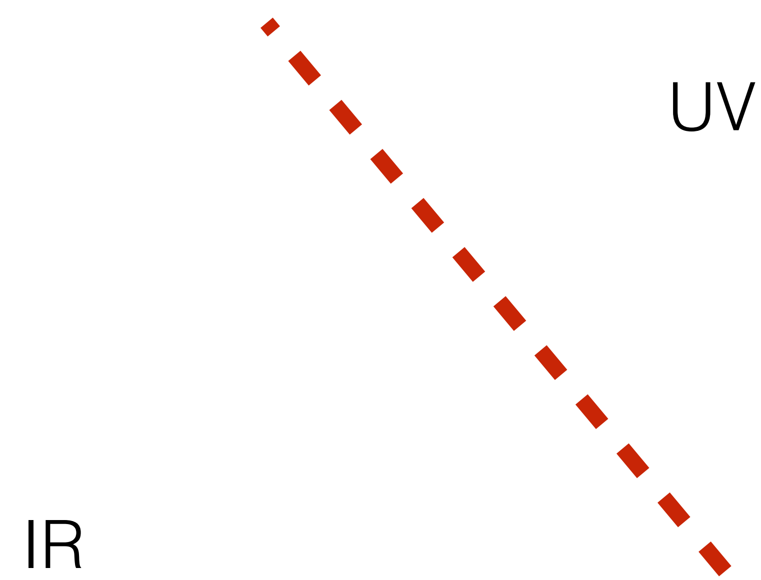
Kim, Sung-Soo

based on a paper with Futoshi Yagi
[arXiv:1411.7903]

5d gauge theory

$$S \sim \int d^5x \, \frac{1}{g_{YM}^2} F^2 \qquad\qquad [g_{YM}^2] = L$$

Non-renormalizable & cutoff theory



High Energy Physics – Theory

Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics

Nathan Seiberg

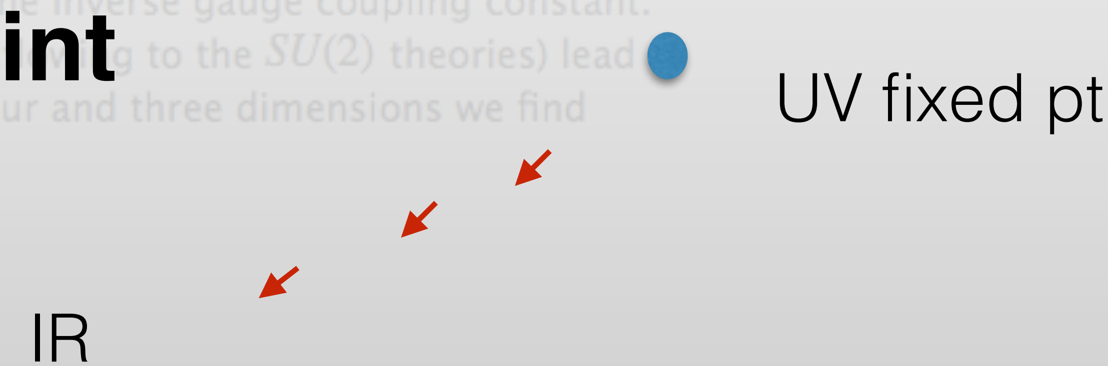
(Submitted on 16 Aug 1996 (v1), last revised 3 Sep 1996 (this version, v2))

We study (non-renormalizable) five dimensional supersymmetric field theories. The theories are parametrized by quark masses and a gauge coupling. We derive the metric on the Coulomb branch exactly. We use stringy considerations to learn about new non-trivial interacting field theories with exceptional global symmetry E_n (E_8, E_7, E_6, $E_5 = Spin(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$, $E_2 = SU(2) \times U(1)$ and $E_1 = SU(2)$). Their Coulomb branch is $\mathbf{R}^+$ and [illegible] operators of these theories leads to a flow to $SU(2)$ gauge theories with $N_f = n - 1$ flavors. In terms of these $SU(2)$ IR theories this relevant parameter is the inverse gauge coupling constant. Other relevant operators [illegible] to the $SU(2)$ theories) lead to flows between them. Upon further compactifications to four and three dimensions we find new fixed points with exceptional symmetries.

Comments: 13 pages, uses harvmac, one reference added
Subjects: High Energy Physics – Theory (hep-th)
Journal reference: Phys.Lett.B388:753-760,1996

Seiberg : Some SUSY theories are well defined

$\exists$ UV fixed point

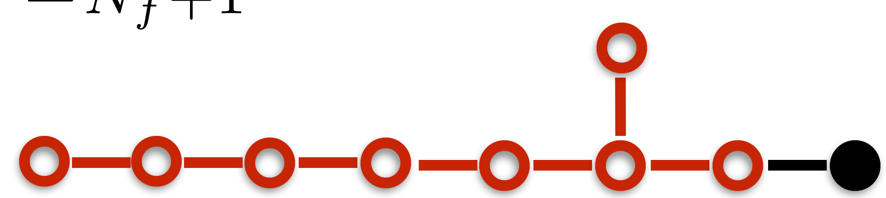


N=1 (8 supercharges) SU(2) gauge theory with N_f flavors (N_f=0,1,…,7)

- N=2 in 4 dim or N=(1,0) in 6 dim.
- Particle content
  - vector multiplet
  - hypermultiplet ($N_f \leq 7$ fundamental flavors)
- non-trivial superconformal fixed point at UV (strong coupling limit)
- In IR, global symmetry is $SO(2N_f) \times U(1)_I$

  conserved U(1) charges
- At UV fixed point, the global symmetry is enhanced

$$SO(2N_f) \times U(1)_I \subset E_{N_f+1}$$



En symmetry and PA3-1 session…

- Superconformal index captures En

[H.-C. Kim, SSK, Lee '12] [Hwang, Joonho Kim, Seok Kim, Park '14]

—> {Chiung Hwang's talk}

- at the level of Nekrasov Partition function

[Mitev, Pomoni, Taki, Yagi '14] [Joonho Kim, Seok Kim, Lee, Park, Vafa '14]

—> {Futoshi Yagi's talk} {Joonho Kim's talk}

- Topological vertex amplitudes

[Bao, Mitev, Pomoni, Taki,Yagi '13] [Hayashi, Kim, Nishinaka '13] [Hayashi, Zoccarato '14]

In this talk….

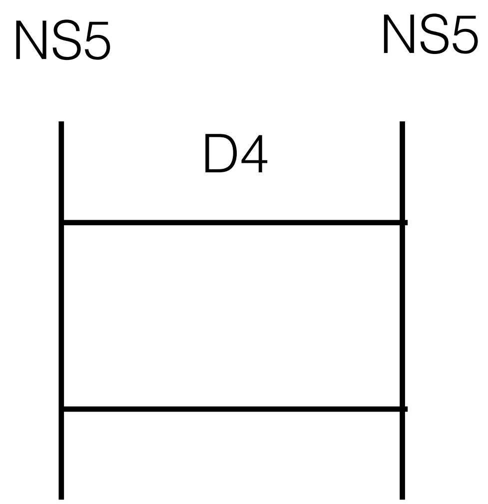
Review 5d SYM using the (p,q) web
Seiberg-Witten curve is E_n invariant

- First half: Brane construction for N_f flavors
- Second half: Based on the brane configuration, compute the SW curve
  - E_{Nf+1} symmetry

    (E_1=SU(2), E_2=SU(2)x U(1), E_3=SU(3) x SU(2), E_4=SU(5), E_5=SO(10), E_6, E_7, E_8)

Brane configuration for pure SU(2) SYM

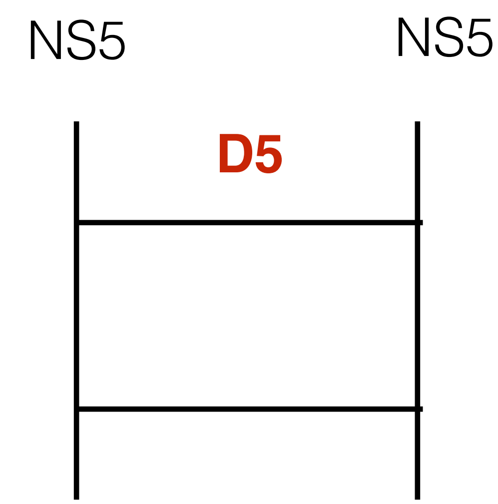
In 4d



	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—				
D4	—	—	—	—			—			

In 5d

naively

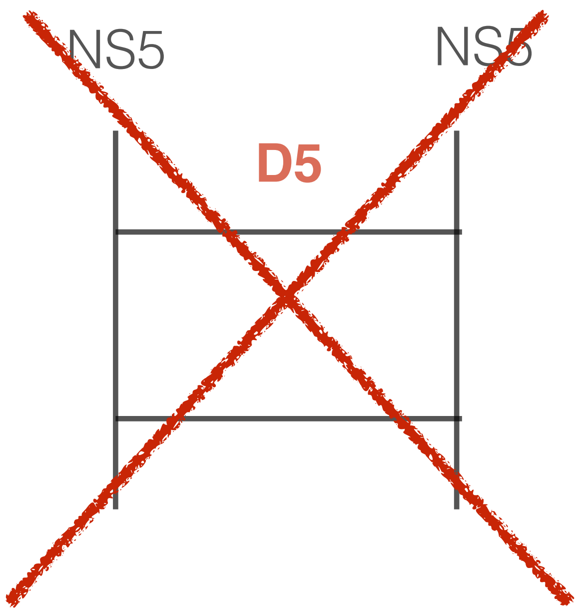


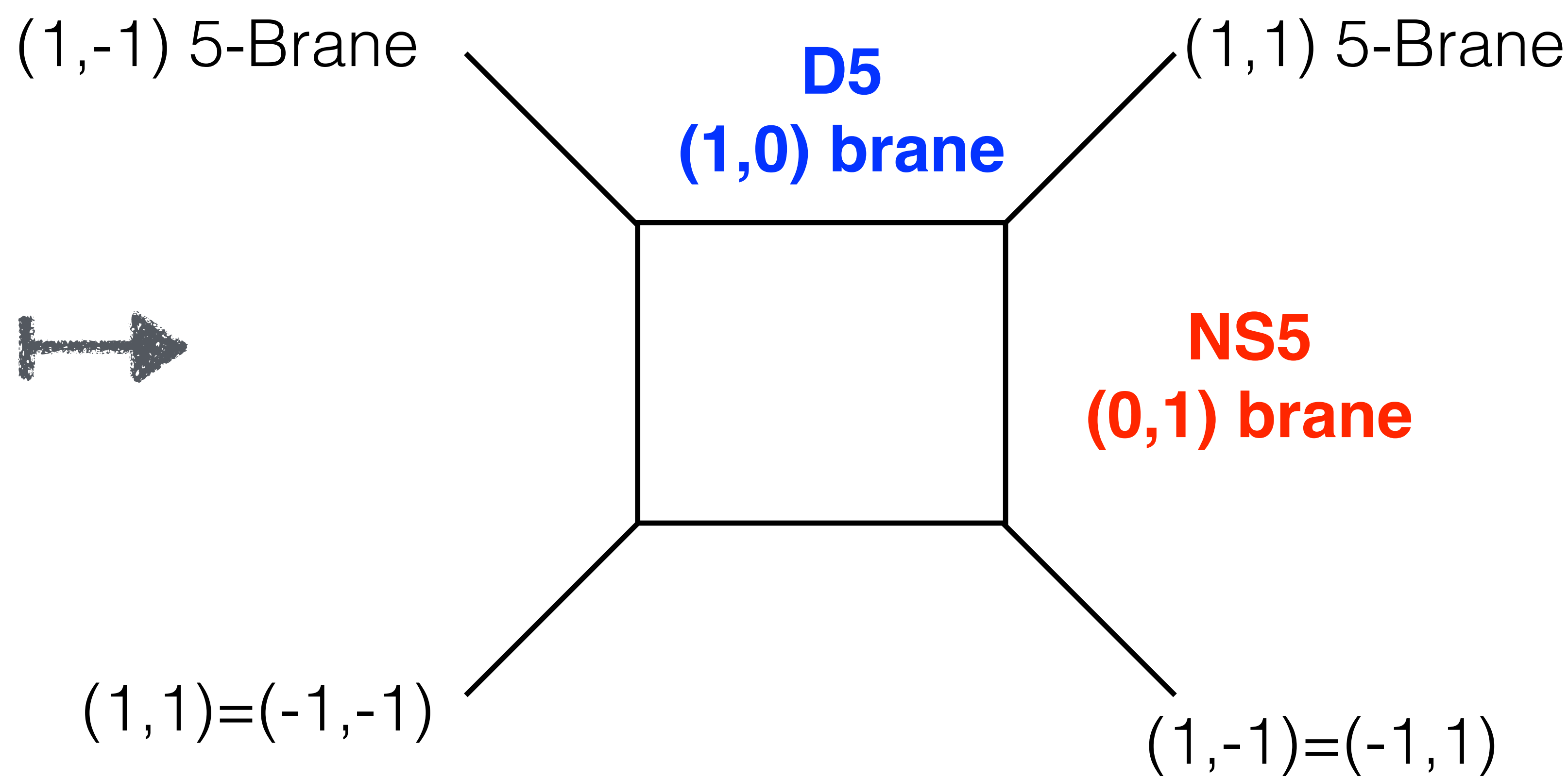
	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—				
D5	—	—	—	—	—		—			

5d SU(2) theory and (p,q) web diagram

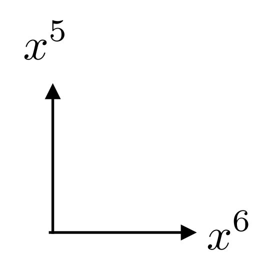
[Aharony-Hanany, '97]

Charge conserving junctions make the (p,q) web





	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(1,1)	–	–	–	–	–	-	-			

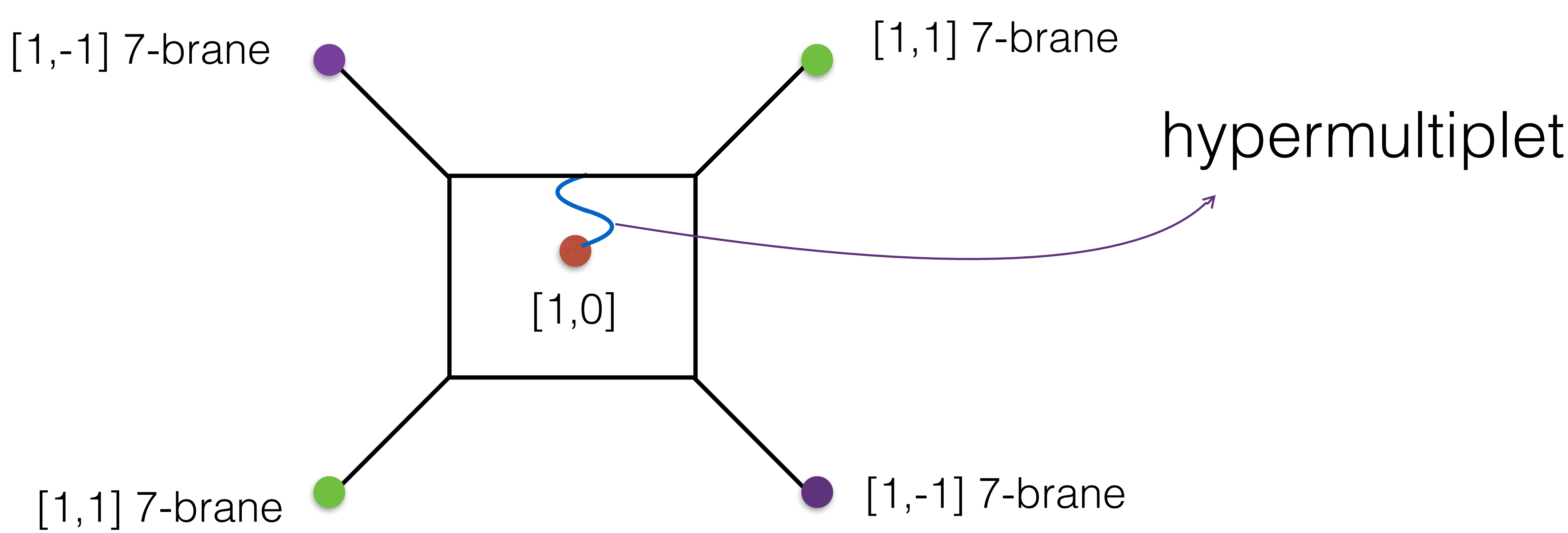


[p,q] 7-branes

(p,q) 5-brane ends on **[p,q]** 7-brane

Flavor= D7 brane = [1,0] 7-brane

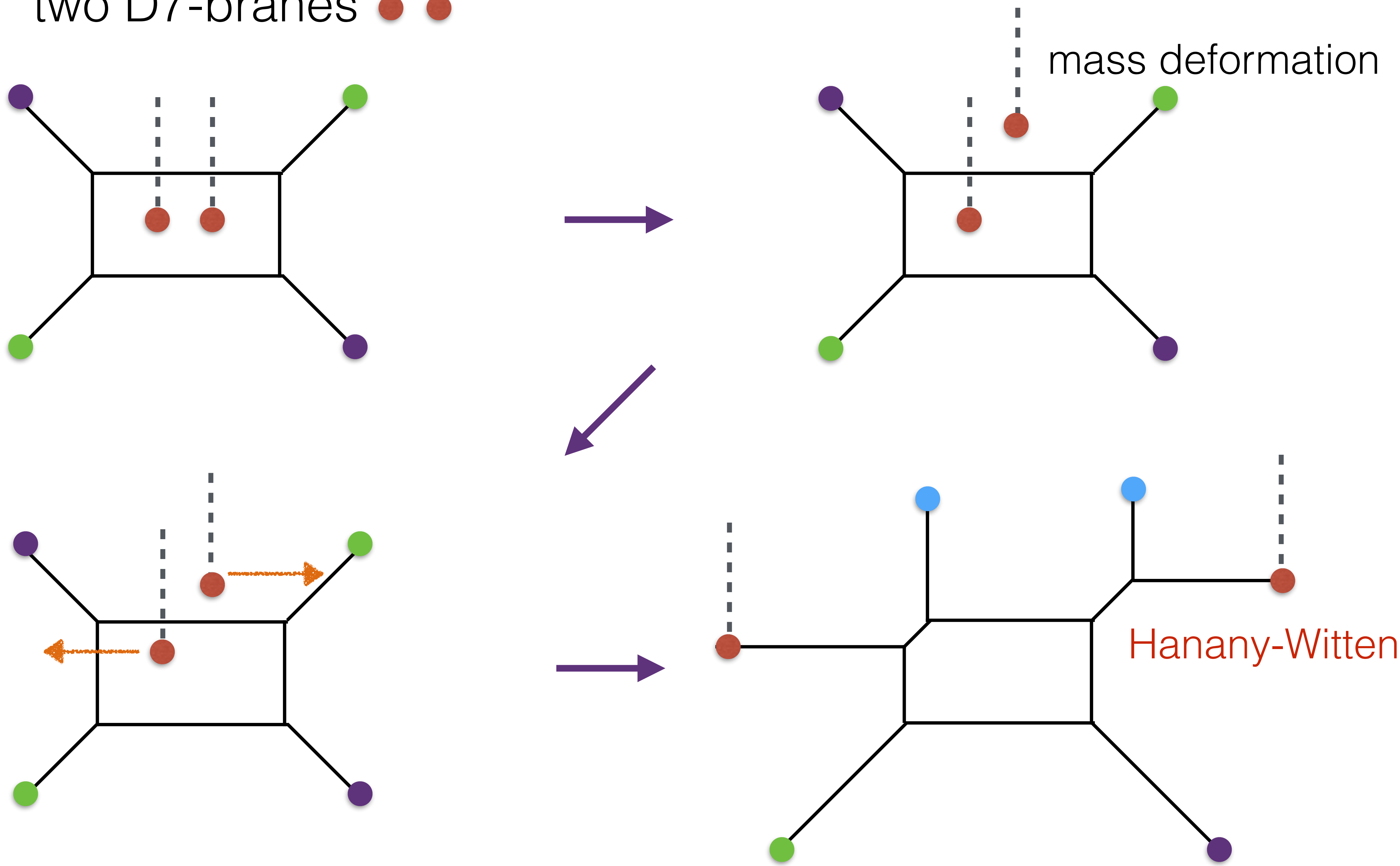
	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–	.			
D5	–	–	–	–	–	.	–			
D7	–	–	–	–	–	.	.	–	–	–



By taking 7-branes to infinity, we get 5-brane (p,q) web again.

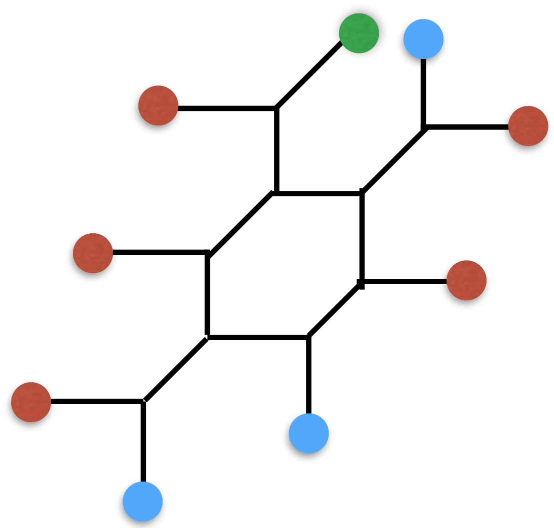
Flavors (7-branes) and Hanany-Witten effects

$N_f = 2$ two D7-branes

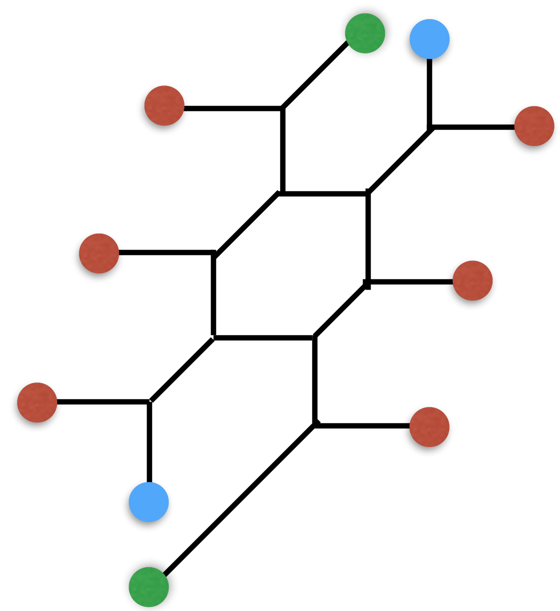


The situation is not very different for other N_f up to 4.

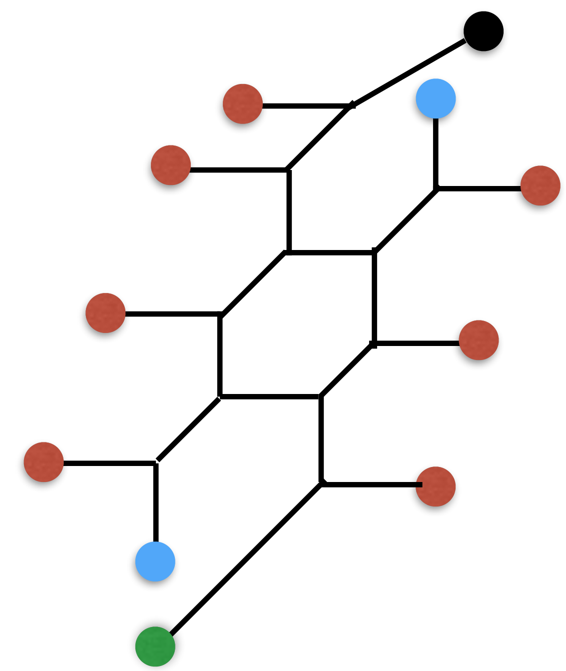
Five, six, seven flavors seem problematic…



$N_f = 5$

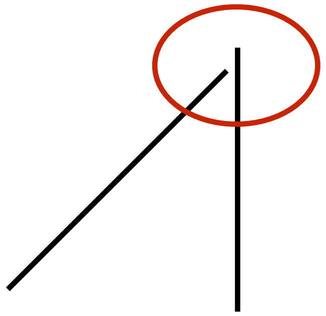


$N_f = 6$



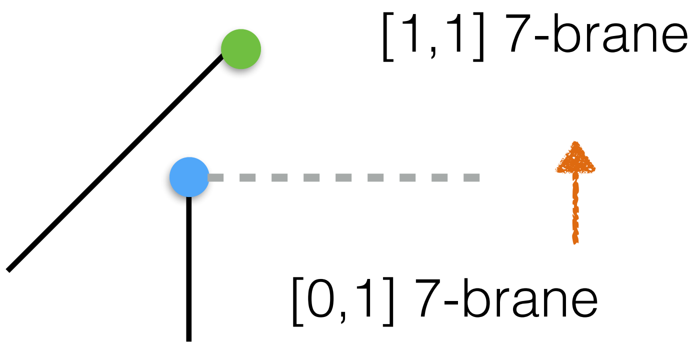
$N_f = 7$

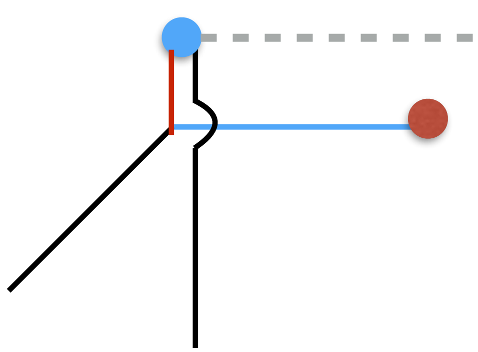
Brane Crossing and Jumping



crossing: - monodromy

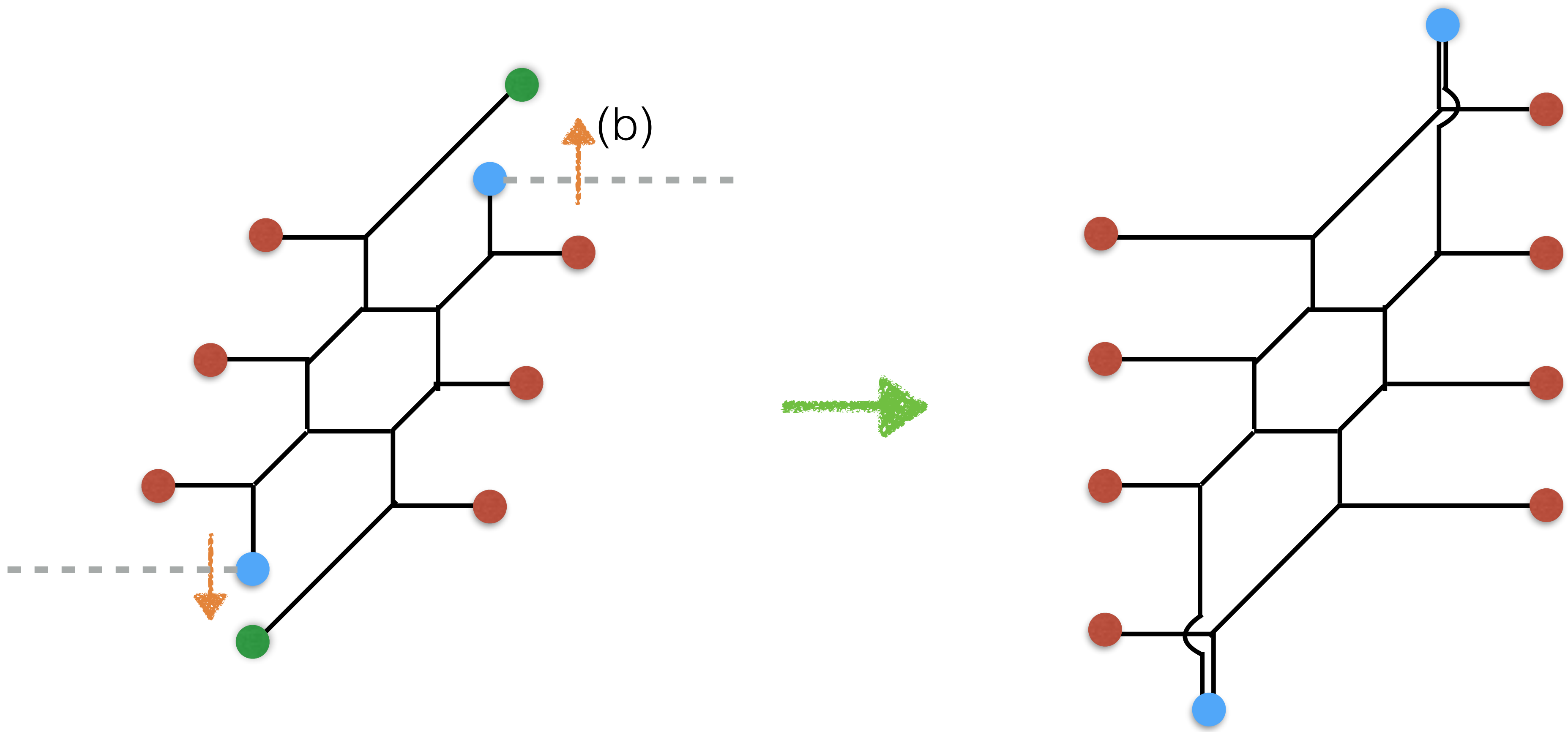
- **Hanany-Witten transition with charge conservation**



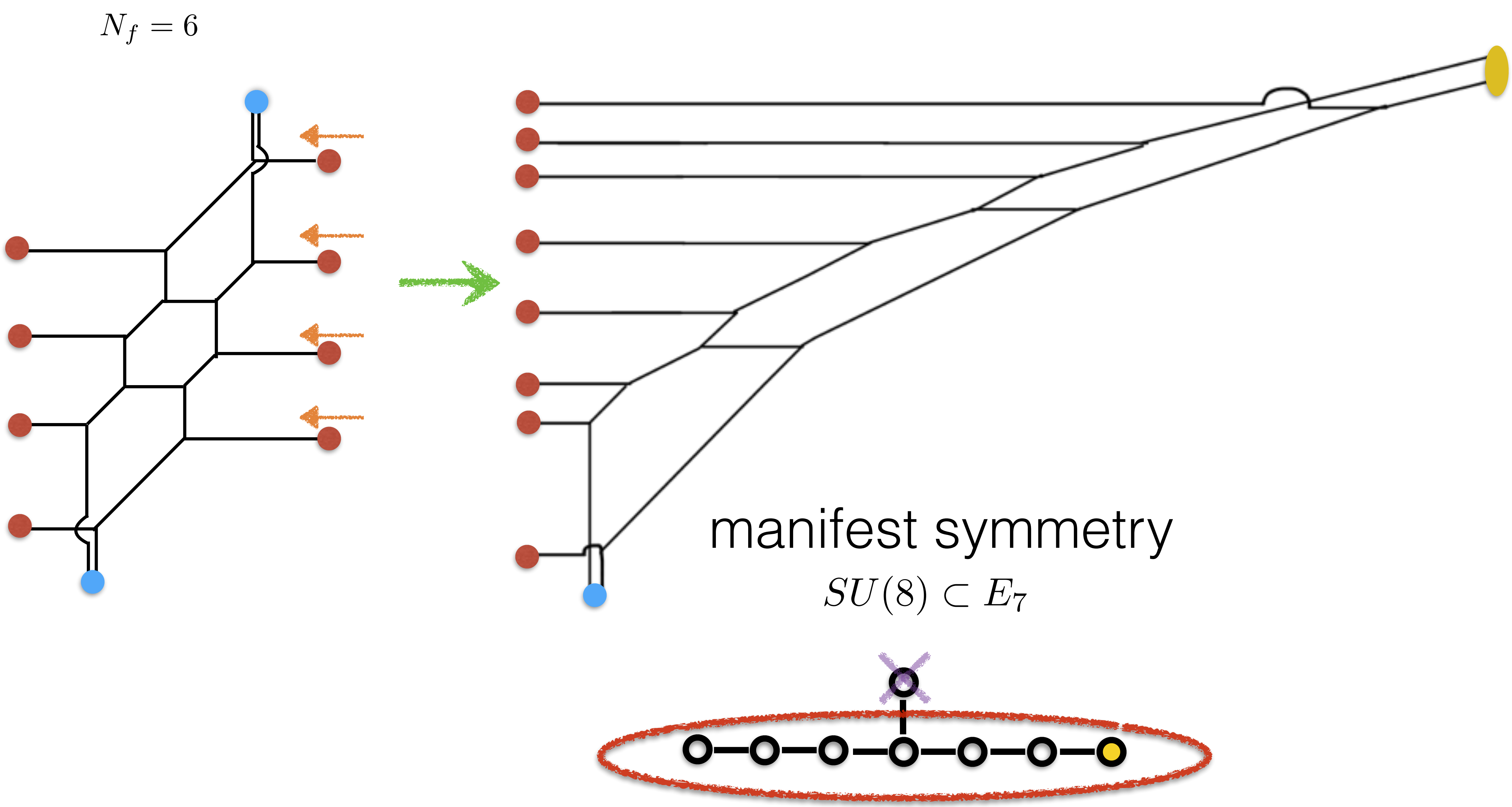


Hanany-Witten

$N_f = 6 \qquad E_7$

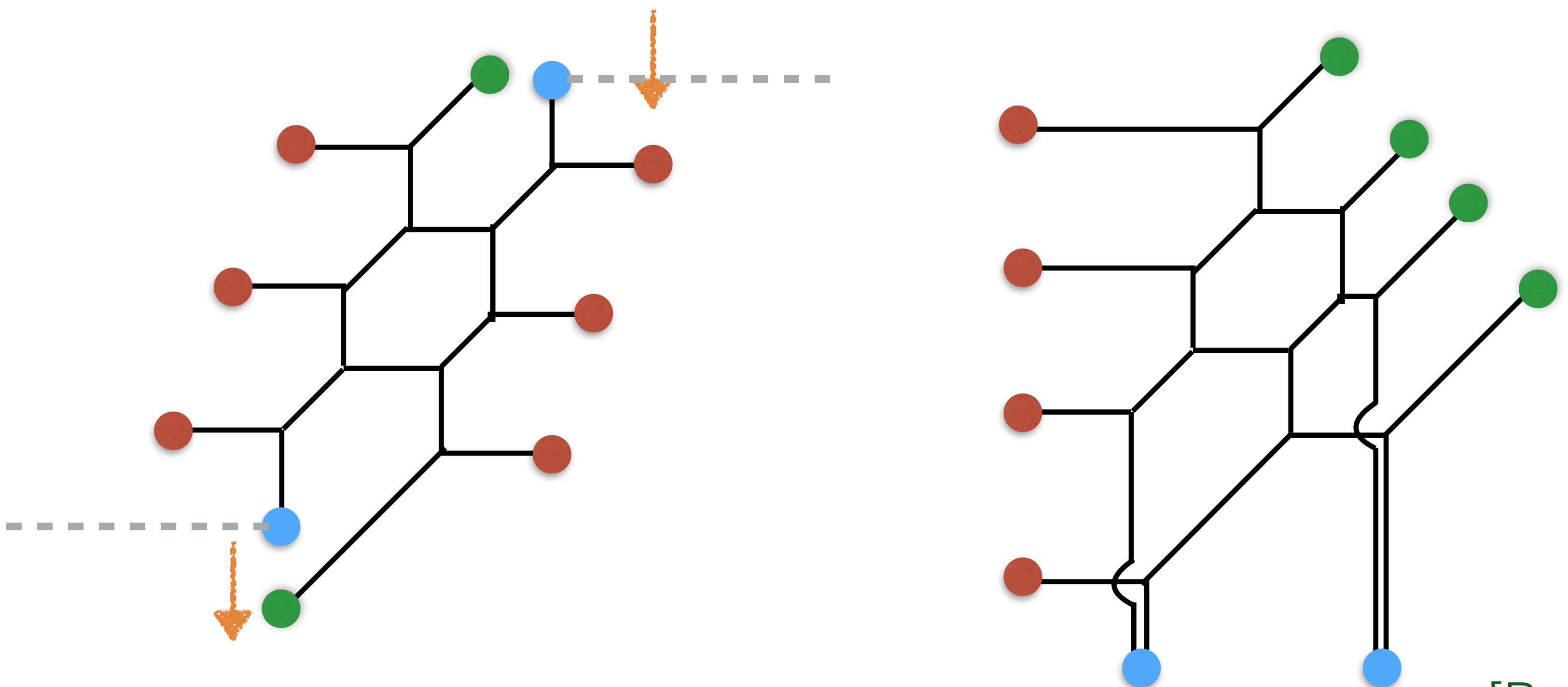


Hanany-Witten effects change the charge of 7-brane



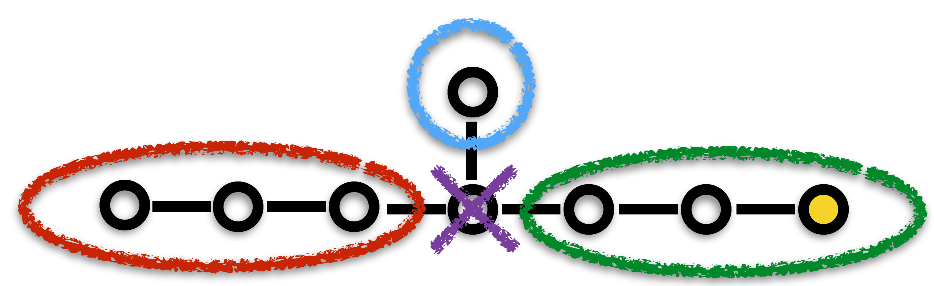
$N_f = 6$
manifest symmetry
$SU(8) \subset E_7$

$N_f = 6$ Many other Hanany-Witten transitions reveal…



[Benini, Benvenutti, Tachikawa '09]

Tuned T_4: $SU(4) \times SU(4) \times SU(2) \subset E_7$

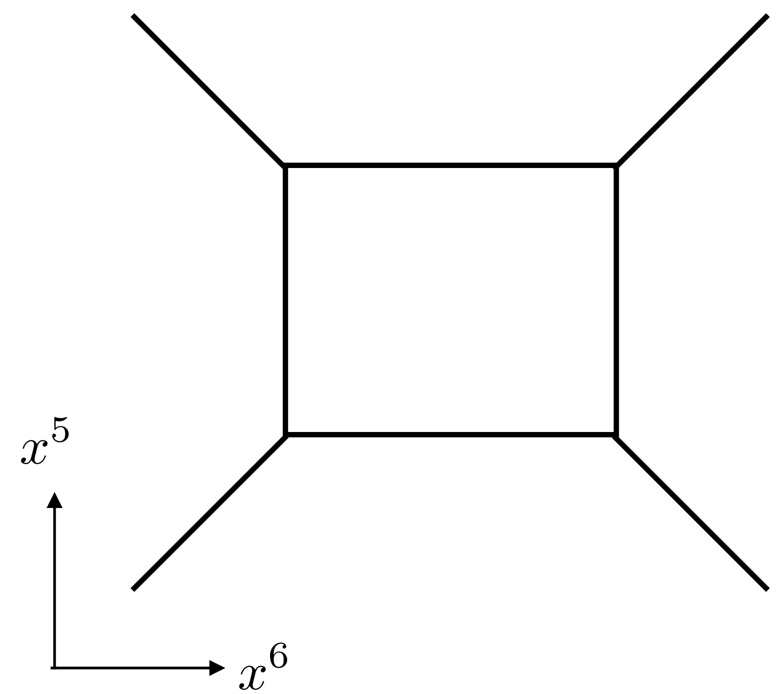


Seiberg-Witten curve

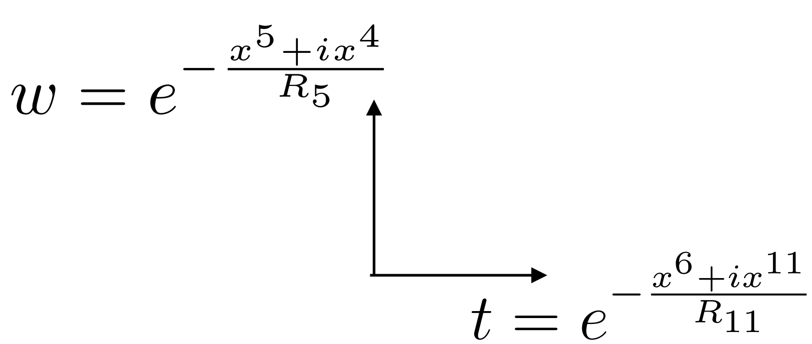
Seiberg-Witten curve :

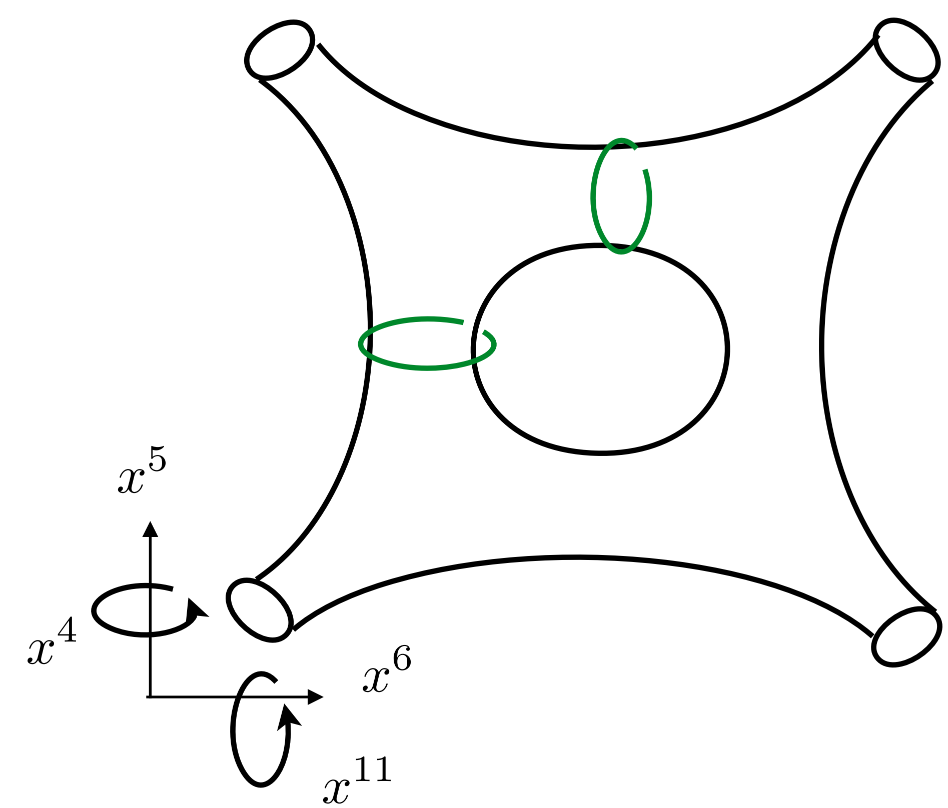
M-theory configuration expressed as an elliptic curve

[Witten '97]



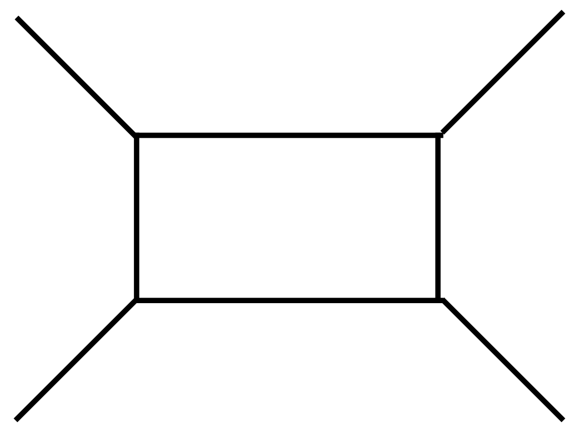
SW curve = **Single M5 brane configuration**
on R^2 x T^2 (x^4,x^5,x^6, x^{11}) space

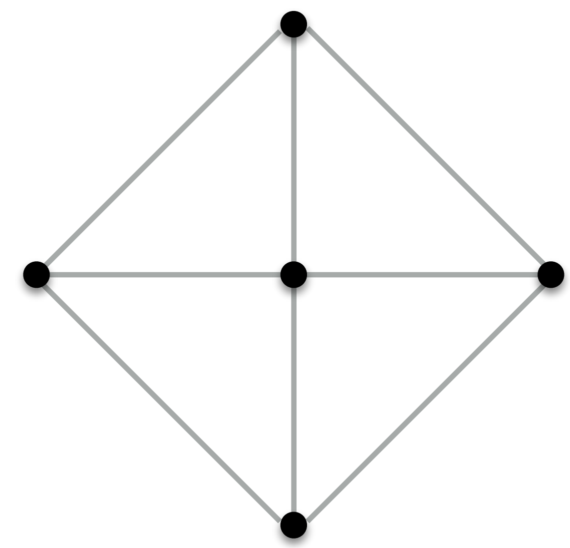




Dual diagram = grid diagram = toric diagram

[Aharony-Hanany-Kol '97] [Leung-Vafa '97]

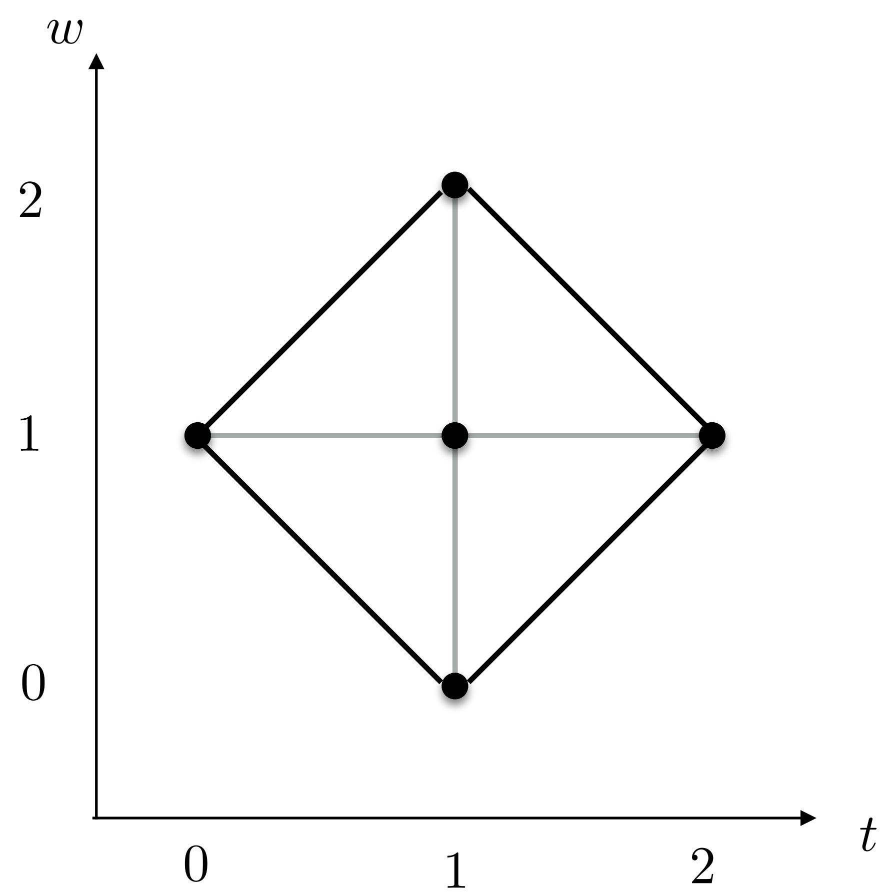




Face	**dot**
Edge	**line connecting dots**
Vertex	**Triangle**

dim(Coulomb branch) = #(internal points)

Toric diagram and SW curve



$$\sum_{dots} c_{ij} t^i w^j = 0.$$

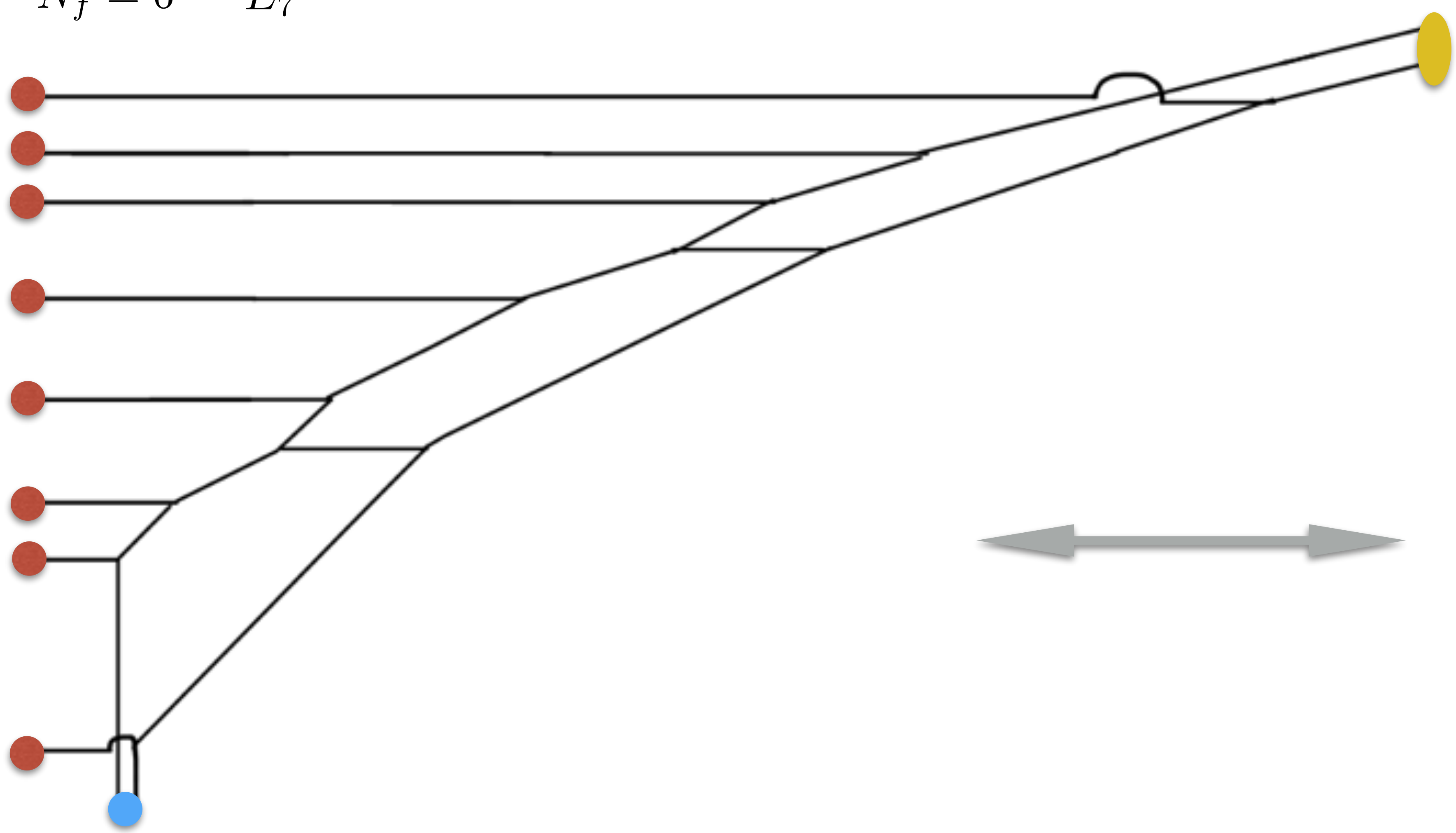
$$c_{01} w + c_{10} t + c_{11} t w + c_{12} t w^2 + c_{21} t^2 w = 0$$

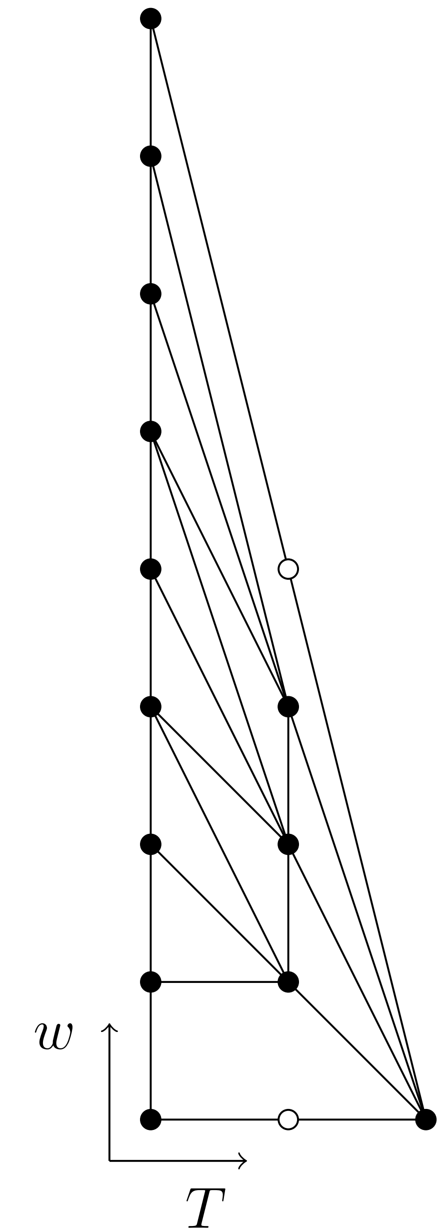
c_{ij} non-zero coefficients

We generalize this systematic way to the cases with N_F flavors

(p,q) web and Dual diagram

$N_f = 6 \qquad E_7$



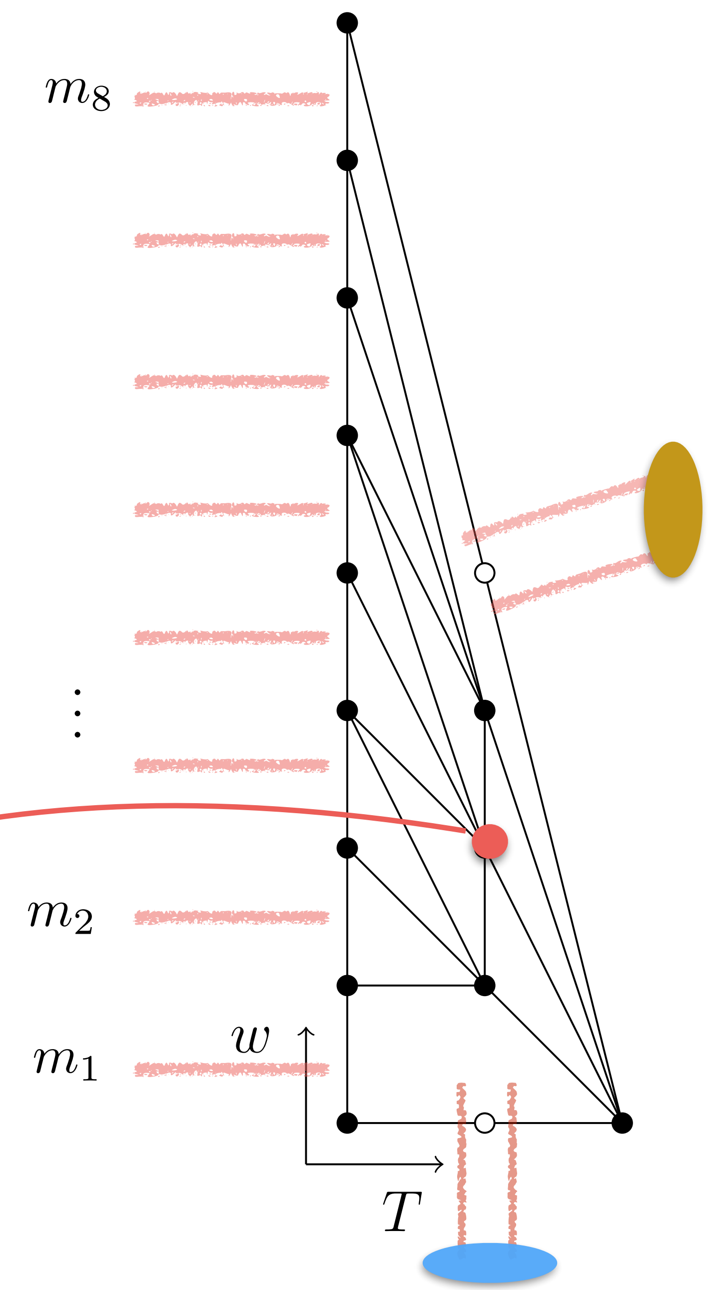


$$T^2 + (-2w^4 + \chi_{\mu_1} w^3 + U\, w^2 + \chi_{\mu_7} w - 2)T$$
$$+ w^8 - \chi_{\mu_1} w^7 + \chi_{\mu_2} w^6 - \chi_{\mu_3} w^5 + \chi_{\mu_4} w^4 - \chi_{\mu_5} w^3 + \chi_{\mu_6} w^2 - \chi_{\mu_7} w + 1 = 0.$$

So far, SU(8) manifest, but …

Toric-like diagram (white dots)

$N_f = 6$



Coulomb modulus parameter

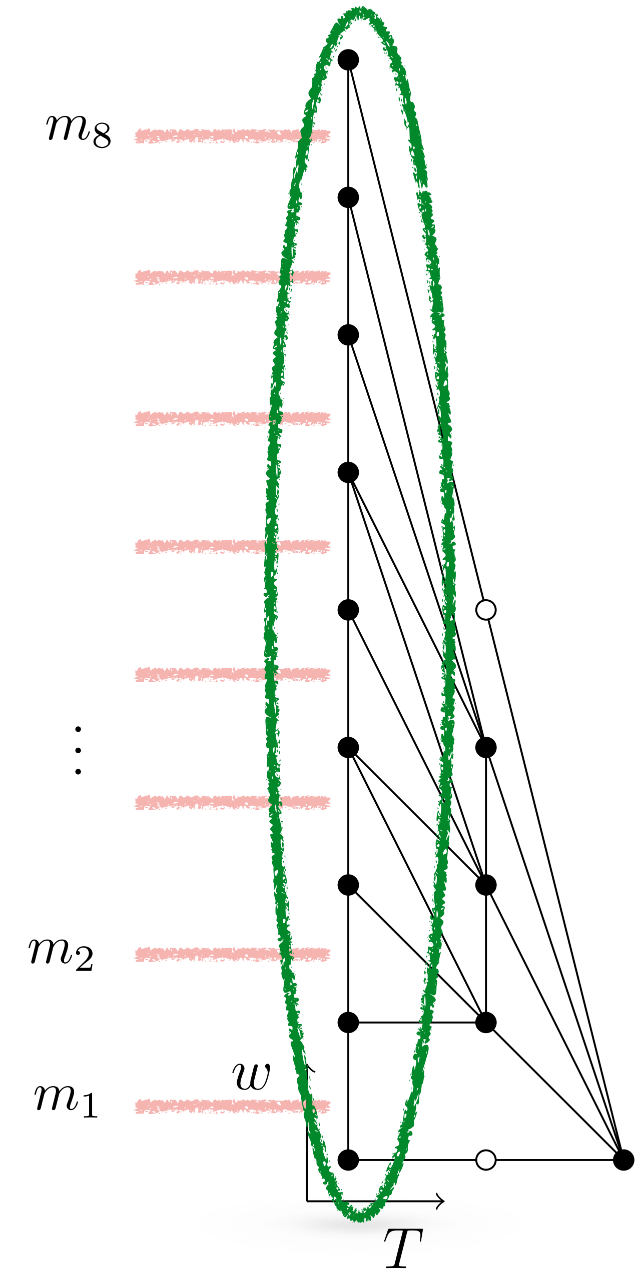
$$T^2 + (-2w^4 + \chi_{\mu_1} w^3 + U\, w^2 + \chi_{\mu_7} w - 2)T$$
$$+ w^8 - \chi_{\mu_1} w^7 + \chi_{\mu_2} w^6 - \chi_{\mu_3} w^5 + \chi_{\mu_4} w^4 - \chi_{\mu_5} w^3 + \chi_{\mu_6} w^2 - \chi_{\mu_7} w + 1 = 0.$$

Toric-like diagram (white dots)

$N_f = 6$

$$(w - m_1)(w - m_2)\cdots(w - m_8)$$

$$\prod_1^8 m_i = 1$$

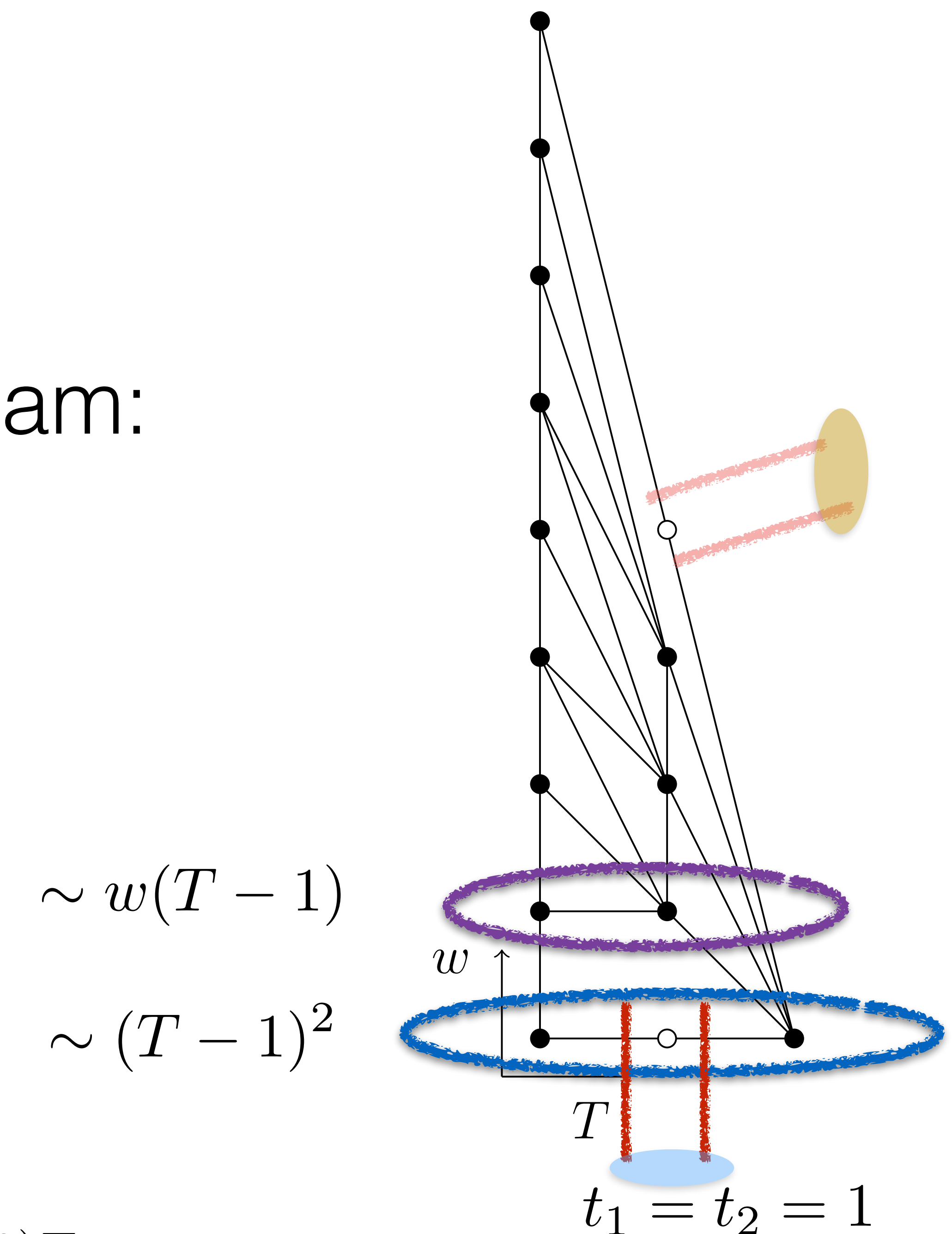


$$T^2 + (-2w^4 + \chi_{\mu_1} w^3 + U\, w^2 + \chi_{\mu_7} w - 2)T$$
$$+ \boxed{w^8 - \chi_{\mu_1} w^7 + \chi_{\mu_2} w^6 - \chi_{\mu_3} w^5 + \chi_{\mu_4} w^4 - \chi_{\mu_5} w^3 + \chi_{\mu_6} w^2 - \chi_{\mu_7} w + 1} = 0.$$

Toric-like diagram (white dots)

$N_f = 6$

Property of toric-like diagram:
degenerate polynomials



$$T^2 + (-2w^4 + \chi_{\mu_1} w^3 + U w^2 + \chi_{\mu_7} w - 2)T$$
$$+ w^8 - \chi_{\mu_1} w^7 + \chi_{\mu_2} w^6 - \chi_{\mu_3} w^5 + \chi_{\mu_4} w^4 - \chi_{\mu_5} w^3 + \chi_{\mu_6} w^2 - \chi_{\mu_7} w + 1 = 0.$$

SU(8) manifest but E₇ invariant

$N_f = 6 \quad SU(8) \subset E_7$

$$T^2 + (-2w^4 + \chi_{\mu_1} w^3 + U w^2 + \chi_{\mu_7} w - 2)T$$
$$+ w^8 - \chi_{\mu_1} w^7 + \chi_{\mu_2} w^6 - \chi_{\mu_3} w^5 + \chi_{\mu_4} w^4 - \chi_{\mu_5} w^3 + \chi_{\mu_6} w^2 - \chi_{\mu_7} w + 1 = 0.$$

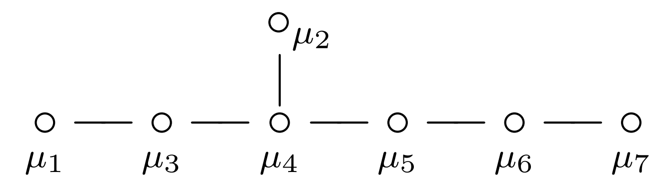
Its j-invariant agrees with E_7 manifest curve:

[Eguchi-Sakai]

$$\begin{aligned} y^2 =& 4x^3 + (-u^2 + 4\chi^{E_7}_{\mu_1} - 100)x^2 + \Big((2\chi^{E_7}_{\mu_2} - 12\chi^{E_7}_{\mu_7})u + 4\chi^{E_7}_{\mu_3} - 4\chi^{E_7}_{\mu_6} - 64\chi^{E_7}_{\mu_1} + 824\Big)x \\ &+ 4u^4 + 4\chi^{E_7}_{\mu_7}u^3 + (4\chi^{E_7}_{\mu_6} - 8\chi^{E_7}_{\mu_1} + 92)u^2 + (4\chi^{E_7}_{\mu_5} - 4\chi^{E_7}_{\mu_1}\chi^{E_7}_{\mu_7} - 20\chi^{E_7}_{\mu_2} + 116\chi^{E_7}_{\mu_7})u \\ &+ 4\chi^{E_7}_{\mu_4} - \chi^{E_7}_{\mu_2}\chi^{E_7}_{\mu_2} + 4\chi^{E_7}_{\mu_1}\chi^{E_7}_{\mu_1} - 40\chi^{E_7}_{\mu_3} + 36\chi^{E_7}_{\mu_6} + 248\chi^{E_7}_{\mu_1} - 2232. \end{aligned}$$

E_7 character decomposition into SU(8)

E_7 Dynkin diagram



$$
\begin{aligned}
\chi_1^{E_7} &= -1 + \chi_1\chi_7 + \chi_4 \\
\chi_2^{E_7} &= \chi_1^2 + \chi_7^2 + \chi_3\chi_7 + \chi_1\chi_5 - 2\chi_2 - 2\chi_6 \\
\chi_3^{E_7} &= 1 - 2\chi_4 + \chi_3\chi_5 + \chi_1^2\chi_6 - 3\chi_2\chi_6 - \chi_1\chi_7 + \chi_1\chi_4\chi_7 + \chi_2\chi_7^2 \\
\chi_4^{E_7} &= -2 + \chi_2^2 - \chi_1\chi_3 + 2\chi_4 - \chi_4^2 + \chi_1^3\chi_5 - 3\chi_1\chi_2\chi_5 + 2\chi_3\chi_5 + \chi_2\chi_5^2 - \chi_1^2\chi_6 \\
&\quad + 3\chi_2\chi_6 + \chi_3^2\chi_6 + \chi_1^2\chi_4\chi_6 - 4\chi_2\chi_4\chi_6 - \chi_1\chi_5\chi_6 + \chi_6^2 + 2\chi_1\chi_7 - \chi_2\chi_3\chi_7 \\
&\quad - \chi_5\chi_7 + \chi_1\chi_3\chi_5\chi_7 - 3\chi_3\chi_6\chi_7 - \chi_2\chi_7^2 + \chi_2\chi_4\chi_7^2 + \chi_3\chi_7^3 \\
\chi_5^{E_7} &= \chi_3^2 + \chi_1^2\chi_4 - 3\chi_2\chi_4 - \chi_1\chi_5 + \chi_5^2 + \chi_1\chi_3\chi_6 - 3\chi_4\chi_6 - \chi_3\chi_7 + \chi_2\chi_5\chi_7 + \chi_4\chi_7^2 \\
\chi_6^{E_7} &= -1 + \chi_1\chi_3 - 2\chi_4 + \chi_2\chi_6 + \chi_5\chi_7 \\
\chi_7^{E_7} &= \chi_2 + \chi_6
\end{aligned}
$$

Conclusion and future direction

- Brane construction of 5d SU(2) gauge theory with N flavors (N<8) : (p,q) web diagram
- From its dual (toric-like) diagram,
  - we computed the Seiberg-Witten curve
  - showed the SW curve is E_{N+1} invariant
- E_{N+1} symmetry is realized as various SU subgroups in the web diagram
- N=8; Hanany-Witten transitions never stop; [in progress]
  outwardly spiral web-diagram
  Affine E_8 symmetry is expected; E-strings
  —> {Joonho Kim's talk}
- N=9 or higher; shrinking spiral web diagrams
  Sign of Landau poles